The procedures for precision dielectric and loss measurements have been developed as described, allowing the construction of the two groups of simulated biotissues. The formulations for the properties of high-water-content materials are valuable for finding phantom materials for many parts of the human body which have a wide range of dielectric properties.

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Observation and Application of High-Order Azimuthal Modes in a Fabry–Perot Resonator

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Abstract—**High-order azimuthal modes ($p = 0, l > 20$) were excited at 10 GHz in a Fabry–Perot resonator. The energy of these modes is distributed in a ring around an energy-free central axis, which lends itself to safety applications on rotating machinery and in measuring density profiles of cylindrical plasmas.**

Introduction

High-order modes in Fabry–Perot resonators have been reported both at laser and at microwave frequencies [1], [2]. Generally, the higher the mode order the greater the radial extent of the beam on the mirror, and therefore a higher energy loss, so that in conventional applications of lasers and beam waveguides these modes are considered undesirable. In this short paper we report on high-order azimuthal modes observed at 10 GHz and describe a device which is based on such high-order modes.

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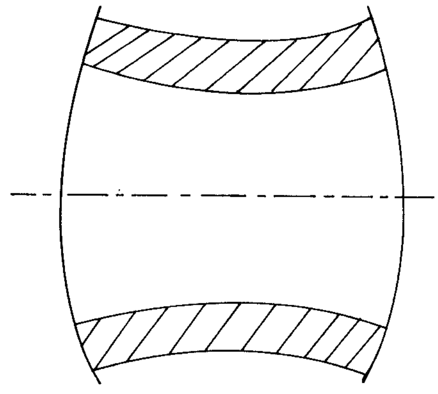

Fig. 1. Cross-sectional representation of the energy distribution of $l = 25$ mode in the $\phi = 0$–$180°$ plane. Boundaries of the shaded area are contours of constant intensity equal to e^{-2} times the peak value in the midplane.

Theory

In the large aperture approximation [1] the amplitude of a component of the electric field is given as (cylindrical coordinates r, z, ϕ)

$$\psi_p{}^l = \left(\sqrt{2}\,\frac{r}{w}\right)^l L_p{}^l\left(\frac{2r^2}{w^2}\right)\frac{w_0}{w}\exp(-r^2/w^2)\cos l\phi \cdot \exp\{j[(2p+l+1)\tan^{-1} az - r^2/w^2az]\} \quad (1)$$

where p and l are the radial and azimuthal mode numbers, respectively, and the remaining parameters are defined as follows:

$w_0{}^2$ $(\lambda/2\pi)[d(2R-d)]^{1/2}$;
d spacing between mirrors;
R radius of curvature of the mirrors;
a $\lambda/\pi w_0{}^2$;
w $w_0[1+a^2z^2]^{1/2}$.

If p is restricted to zero the expression in (1) is simplified such that in the midplane of the device the intensity of the field is

$$(\psi_0{}^l)^2 = (2r^2/w_0{}^2)^l \exp(-2r^2/w_0{}^2)\cos^2 l\phi. \quad (2)$$

This function is zero on the axis of the resonator and has $2l$ maxima on a radius $r_{max} = w_0\sqrt{l/2}$. The circumference implied by this radius is just $2\pi w_0\sqrt{l/2}$, so that the distance between adjacent azimuthal maxima is just $\pi w_0/\sqrt{2l}$.

Thus, the higher the value of l the larger the area near the axis which is free of energy and the closer to each other are the adjacent azimuthal maxima. Fig. 1 shows a cross section of energy in the $\phi = 0$–$180°$ plane for $l = 25$.

Experiment

We have excited such modes at X band with l as high as 25. Distribution of mode energy was measured with the absorptive probe method described in [2]. The value of l was determined by using as a probe a thin ceramic rod rigidly fixed perpendicular to a larger shaft on the axis of the device. Each revolution of the shaft swept the ceramic rod through $2l$ maxima.

The mirrors were identical to those in [2], with the exception that the azimuthal modes were excited through coupling irises near the edge of the mirror rather than on the axis. A three-dimensional oscilloscope representation of the energy distribution in the midplane of the device is shown in Fig. 2. Note that with an l of 25 the adjacent maxima are so close together that they are just able to be distinguished in those places on the

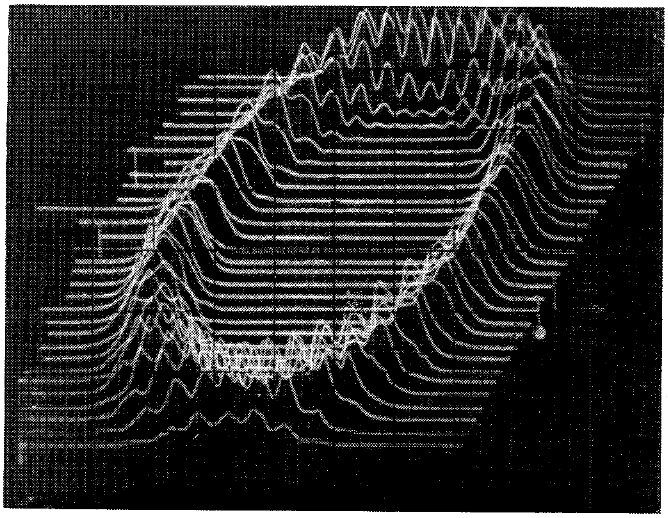

Fig. 2. Three-dimensional oscillographic representation of the energy distribution of $l = 25$ mode in the midplane of the device.

photograph where the probe is traveling tangentially to the ring of maximum intensity. We have utilized this property to use the device as an intrusion sensor, where the energy-free region near the axis contains some type of rotating machinery and appropriate electronics turn off the machine if an operator's hand, for example, comes into the region of resonant energy, thus disturbing the resonance.

Conclusion

High-order azimuthal modes with l as high as 25 have been excited at X band in a Fabry–Perot resonator. The localization of mode energy near the radius of maximum field intensity $r = w_0\sqrt{l/2}$ produced a "curtain" of energy around a central region of energy-free space. This "curtain" has found practical application as an intrusion detector in such areas as machine-operator safety. Although the need for it has not arisen in this laboratory, at least one further application suggests itself: the measurement of density profiles in a cylindrical plasma. The shift in resonant frequency caused by the plasma is a function of the plasma density convoluted with the mode energy. The use of modes of successively larger l would yield a radial density profile rather than an average density obtained with just one mode.

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